quick study

The versatility of nanoscale mechanical resonators

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The vibrations of tiny "diving boards" enable scientists to view the world one atom at a time and may allow the observation of quantum effects in mechanical systems.

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Mechanical oscillators have a long history of assisting scientific pursuit. For example, a desktop torsional balance was a key tool used by Charles Augustin de Coulomb in his late-18th-century studies of electrostatics. Henry Cavendish accomplished his contemporary quantitative measurements of the gravitational force with a mechanically resonant version of a similar torsion balance. Today, mechanically resonant systems range in scale from the very large, such as the 4-km detectors developed for the Laser Interferometer Gravitational-Wave Observatory (see the article by Barry Barish and Rainer Weiss, PHYSICS TODAY, October 1999, page 44), to the very small, including devices that can detect the magnetic force from a single electron spin—perhaps ultimately from a single nuclear spin.

The basic mechanical element for the smallest systems is often a cantilever, a beam that is clamped at one end. To understand how it behaves when fabricated at nanometer dimensions, first consider a more familiar case: a 2-meter long poolside diving board made of fiberglass-coated metal. When a 50-kg diver stands on the free end of the board, the end dips by about 5 cm. Evidently, the diving board has an effective spring constant k of something like 10^4 N/m. After the diver leaps from the end into the water, the diving board vibrates with its mechanical resonance frequency ν, a few hertz. The cantilever's effective mass m_{eff}, defined by the simple harmonic oscillator formula $m_{\mathrm{eff}} \equiv k/(2\pi\nu)^2$, is thus about 100 kg.

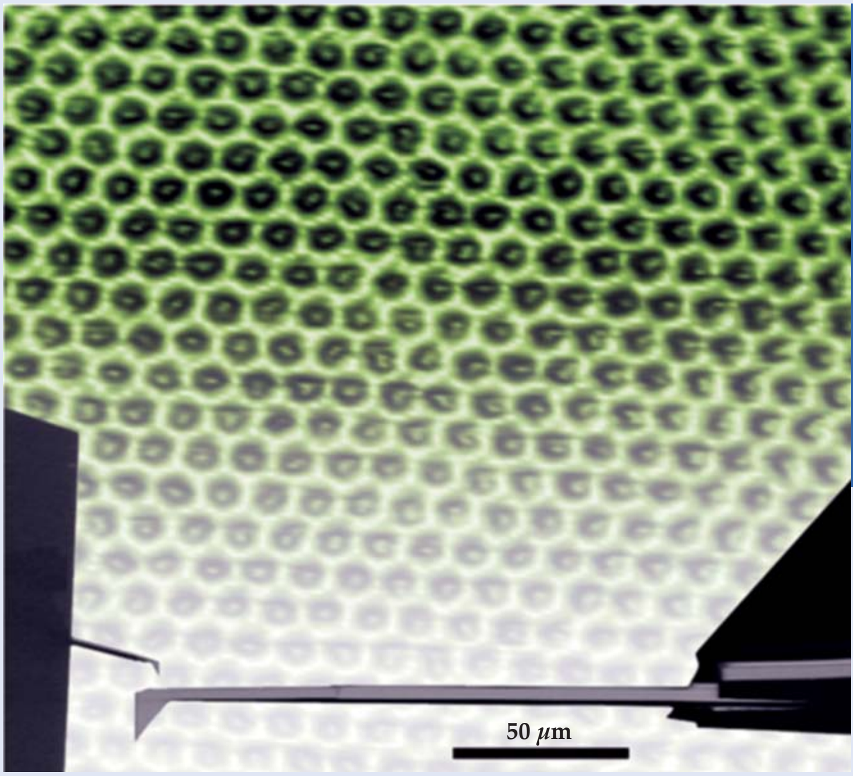
Now imagine scaling all the dimensions of the diving board down by a million, to something like 2 μm long (L), maybe 20 nm thick (t), and 200 nm wide. The effective mass, proportional to the volume, would become a factor of 10^{18} smaller, about 10^{-16} kg. As dictated by the Euler–Bernoulli thin-beam theory, the mechanical resonance frequency, proportional to t/L^2, would increase to a few million hertz. The spring constant $k = m_{\mathrm{eff}}(2\pi\nu)^2$ would be around 10^{-2} N/m. The interest in scaling cantilevers to smaller dimensions immediately becomes apparent: Nanoscale cantilevers have high resonance frequencies, minuscule masses, and small spring constants that yield much larger displacements for a given force.

A look at the atom

A submillimeter-scale cantilever is at the heart of the atomic force microscope, a common analytic instrument that generates images of surfaces with single-atom resolution (see the article by Daniel Rugar and Paul Hansma, PHYSICS TODAY, October 1990, page 23). To achieve that resolution, the AFM's roughly 100-μm-long cantilever must have an extremely sharp tip attached to the free end, as shown in the figure. The experimenter drags the tip over a surface; the resulting minute deflections of the cantilever end can be detected with the help of a laser beam. In another mode of operation, the tip is lifted off the surface. Nonetheless, the tip is still acted on by surface forces—either electrostatic forces from surface charges or van der Waals forces from fluctuations in local dipoles on the surface. Those forces effectively change the spring constant of the cantilever and thus its mechanical resonance frequency. By gently shaking the cantilever while watching its motion, an experimenter can monitor the resonance frequency and thus measure and map out surface forces as a function of tip position. Such force maps allow for surface images that still achieve single-atom resolution.

Electrostatic and van der Waals forces are not the only forces that can be probed with tiny cantilevers. Researchers are developing versions of cantilevers that can be used in magnetic resonance force microscopes, instruments that measure and map out forces from electron or nuclear spins on a surface. If a very small magnet is included in the cantilever construction, the cantilever will respond to the magnetic fields generated by the magnetic moments associated with individual surface spins. In practice, one needs cantilevers with extremely small spring constants; that's achieved by making the cantilever very long and very thin. By optimizing the measurement of cantilever motion, an experimenter can detect the extraordinarily small magnetic moments of a single electron or a few nuclei and then generate an image of the spin density. For example, in a tour de force experiment, Rugar and colleagues from IBM Research detected a single electron spin, achieving a resolution of 25 nm in the scanning direction. Ultimately, the magnetic resonance force microscope may lead to surface maps with single-atom magnetic resolution. Not only would one know the location of an atom, one could also identify the atom through its magnetic moment. Thus, for example, proteins or strands of DNA might be sequenced by direct measurement.

Nanometer-size cantilevers, with their small effective mass, are good tools for inertially "weighing" minute amounts of material. To do that, one adds mass to the cantilever's free end and monitors the resulting change in the resonance frequency. In that manner, Kenneth Jensen, Alex Zettl,



Cantilevers and carbon. A relatively large cantilever such as the one that dominates the foreground is the key component of an atomic force microscope, which is capable of achieving atomic-scale resolution. Indeed, the background photograph is an AFM image of the graphite lattice. To the left in the foreground is an ultralow-mass cantilever about 1/10 the size of those usually used in AFMs. Its high resonance frequency—9 MHz as compared with the 300 kHz of the conventional cantilever—allows for faster imaging and reduces the impact of thermal noise. (Courtesy of Asylum Research, Santa Barbara, CA.)

and colleagues at the University of California, Berkeley, measured the mass of a single gold atom with a cantilever made from a carbon nanotube.

In a second popular type of nanomechanical resonator, a beam is fixed at both ends, rather than at just one as for a cantilever. According to the Euler–Bernoulli thin-beam theory, a doubly clamped beam has a resonance frequency approximately six times that of a cantilever with the same geometry and material; the increased frequency is useful for applications such as timing elements for integrated circuits or filters for telecommunications applications. An additional reason for the popularity of the doubly clamped beam is that it can be easier to fabricate and measure than a comparable cantilevered beam.

Doubly clamped beams are sometimes referred to as violin strings. After all, their appearance and the shape of their resonant modes closely resemble those of a stretched wire. However, the mechanical response of a beam is dominated by the beam's material stiffness, whereas that of a violin string is determined by the external tension. As a result, the mechanics of the two resonators are quite different. Indeed, as with the cantilever, the resonance frequency for the doubly clamped beam is proportional to t/L^2. By contrast, the resonance frequency of a violin string is proportional to $1/L$.

A look at quantum physics

The AFM can measure extremely weak forces. In one particularly interesting experiment, a surface and the tip of an AFM cantilever are coated with metal, kept at the same electrostatic potential, and brought close to one another. At sufficiently small distances, an unusual interaction called the Casimir force becomes detectable. That force is due to the quantum mechanical zero-point energy of the electromagnetic modes localized between the tip and the surface (see PHYSICS TODAY, February 2007, page 40, and May 2007, page 16). Measurements of the Casimir force appear to be in reasonable agreement with those predicted by quantum mechanics, although interesting questions remain.

Researchers are trying to develop other ways in which quantum mechanical effects can be measured in mechanical systems; nanomechanical resonators may provide a means for doing that. The signatures one might look for include the superposition and the interference of the resonator's quantum states. However, a number of challenges must be met if those signatures are to be observed. Thermal effects, which cause the resonator to occupy quantum levels with energies up to a few times the thermal energy (k_BT, with k_B being Boltzmann's constant and T, the temperature), will "smear out" most quantum effects. They can be minimized if $h\nu \gg k_BT$ (h is Planck's constant), which means one wants to work with the highest-frequency resonators and at the lowest possible temperatures. By building resonators with frequencies greater than a gigahertz, or by working at temperatures below a millikelvin, experimenters can satisfy that energy inequality.

Another consideration is that the resonator must retain its energy and its quantum coherence—the relative complex amplitude between two quantum states—long enough that a measurement can be performed. The energy lifetime may be long enough in some mechanical systems, but the coherence time has yet to be measured for any nanomechanical resonator. Perhaps that quest will resonate with some young researcher.

Additional resources

▶ Euler–Bernoulli beam equation, http://en.wikipedia.org/wiki/Euler-Bernoulli_beam_equation.

▶ A. N. Cleland, *Foundations of Nanomechanics: From Solid-State Theory to Device Applications*, Springer, New York (2003), sec. 9.7.

▶ A. Cho, "Researchers Race to Put the Quantum into Mechanics," *Science* **299**, 36 (2003).

▶ D. Rugar et al., "Single Spin Detection by Magnetic Resonance Force Microscopy," *Nature* **430**, 329 (2004).

▶ S. Chao et al., "Nanometer-scale Magnetic Resonance Imaging," *Rev. Sci. Instrum.* **75**, 1175 (2004).

▶ K. L. Ekinci, X. M. H. Huang, M. L. Roukes, "Ultrasensitive Nanoelectromechanical Mass Detection," *Appl. Phys. Lett.* **84**, 4469 (2004).

▶ K. Jensen, K. Kim, A. Zettl, "An Atomic-Resolution Nanomechanical Mass Sensor," *Nature Nanotech.* **3**, 533 (2008). ■